TOC: Assignment 4

Due on: 02 Nov

1. Consider the following language:

$$\{\langle M\rangle \mid L(M) \text{ is regular}\}$$

   (a) Is it Recursively Enumerable?

   (b) Is it Co-Recursively Enumerable?

   Justify.

2. A *linear bounded automaton* (LBA) is exactly like a one-tape Turing machine, except that the input string $x \in \Sigma^*$ is enclosed in left and right endmarkers $\vdash$ and $\dashv$ which may not be overwritten, and the machine is constrained never to move left of the $\vdash$ nor right of the $\dashv$. It may read and write all it wants between the endmarkers.

   (two-way, read/write)

   (a) Give a rigorous formal definition of deterministic linearly bounded automata, including a definition of configurations and acceptance. Your definition should begin as follows: "A deterministic linearly bounded automaton (LBA) is a 9-tuple

$$M = (Q, \Sigma, \Gamma, \vdash, \dashv, \delta, s, t, r)$$

   where Q is a finite set of states. ..."

   (b) Let M be a linear bounded automaton with state set Q of size k and tape alphabet Γ of size m. How many possible configurations are there on input x, $|x| = n$?

   (c) Argue that the halting problem for deterministic linearly bounded automata is decidable. (*Hint:* You need to be able to detect after a finite time if the machine is in an infinite loop. Presumably the result of part (b) would be useful here.)

   (d) Prove by diagonalization that there exists a recursive set that is not accepted by any LBA.